

Chapters IV and V describe quantum properties of an ensemble of photons.

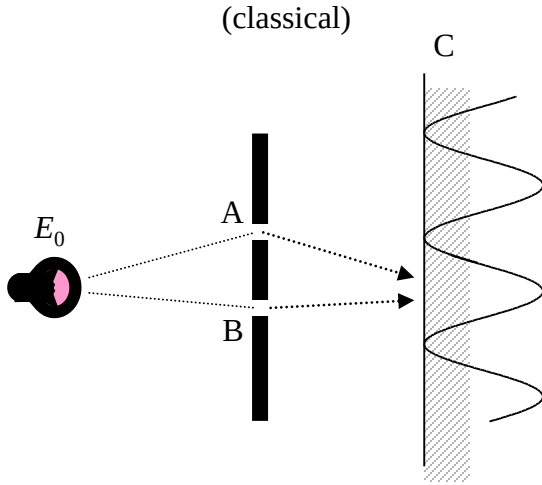
This chapter treats one or two photons.

One-photon interference

A quantum state is expressed by a superposition (i.e., linear combination) of basis states with coefficients (i.e., probability amplitudes) of complex numbers.

The probability amplitude is equivalent to the field amplitude in classical optics.

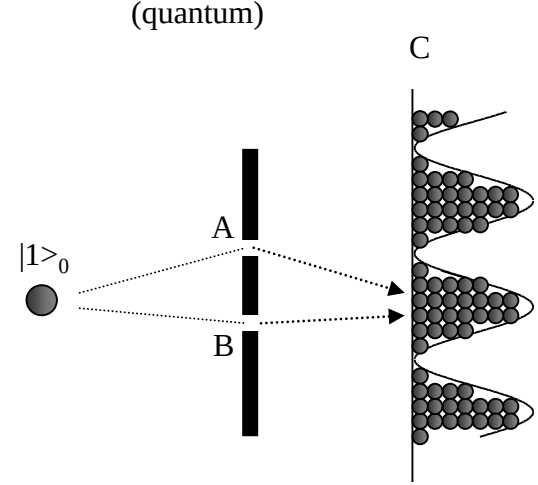
Subsequently, an interference phenomenon similarly occurs in the quantum and classic worlds.



$$\begin{aligned} E_0 &\rightarrow E_a \mathbf{e}_a e^{ikL_{a0}} + E_b \mathbf{e}_b e^{ikL_{b0}} \\ &\rightarrow E_a \mathbf{e}_c e^{ikL_{ca0}} + E_b \mathbf{e}_c e^{ikL_{cb0}} \\ &= (E_a e^{ikL_{ca0}} + E_b e^{ikL_{cb0}}) \mathbf{e}_c \end{aligned}$$

light intensity

$$\begin{aligned} &\left| E_a e^{ikL_{ca0}} + E_b e^{ikL_{cb0}} \right|^2 \\ &= |E_a|^2 + |E_b|^2 + 2|E_a||E_b| \cos(k\Delta L + \Delta\theta) \end{aligned}$$



$$\begin{aligned} |1>_0 &\rightarrow a e^{ikL_{a0}} |1>_a + b e^{ikL_{b0}} |1>_b \\ &\rightarrow a e^{ikL_{ca0}} |1>_c + b e^{ikL_{cb0}} |1>_c \\ &= (a e^{ikL_{ca0}} + b e^{ikL_{cb0}}) |1>_c \end{aligned}$$

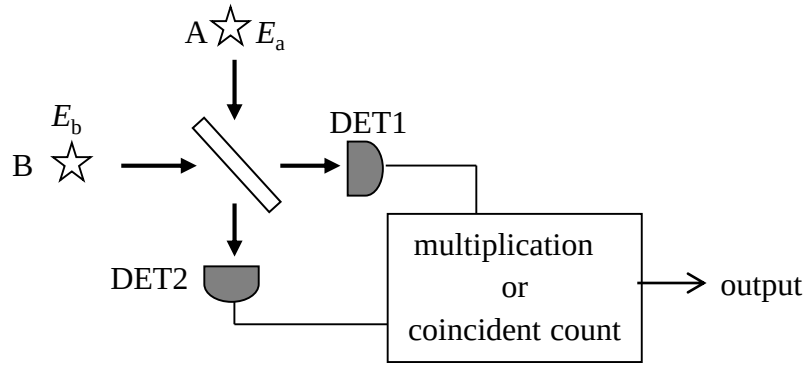
probability of photon detection

$$\begin{aligned} &\left| a e^{ikL_{ca0}} + b e^{ikL_{cb0}} \right|^2 \\ &= |a|^2 + |b|^2 + 2|a||b| \cos(k\Delta L + \Delta\theta) \end{aligned}$$

$$\left(\begin{array}{l} \Delta L = L_{ca0} - L_{cb0} \\ \Delta\theta: \text{phase difference between } E_a \text{ and } E_b \end{array} \right)$$

However, interference occurs differently in two-photon cases.

Lights are incident to a beam splitter from different ports, and multiplication of the intensities or coincident counts at the two output ports is measured.



(classical)

$$\begin{aligned}
 & |irE_a + tE_b|^2 \times |tE_a + irE_b|^2 \quad \left(\begin{array}{l} t: \text{amplitude transmittance} \\ r: \text{amplitude reflectance} \\ \text{phase shift of } \pi/2 \text{ between transmission and reflection} \end{array} \right) \\
 &= \{|rE_a|^2 + |tE_b|^2 + itrE_aE_b^* + c.c.\} \{|tE_a|^2 + |rE_b|^2 - itrE_aE_b^* + c.c.\} \quad (T = t^2, R = r^2) \\
 &= TR|E_a|^4 + TR|E_b|^4 + R^2|E_a|^2|E_b|^2 + T^2|E_a|^2|E_b|^2 - 2TR|E_a|^2|E_b|^2 + \dots
 \end{aligned}$$

$$\left\{ \begin{array}{l} \text{1st term: \{intensity of light(A} \rightarrow \text{D1)\} \times \{intensity of light(A} \rightarrow \text{D2)\}} \\ \text{2nd term: \{intensity of light(B} \rightarrow \text{D1)\} \times \{intensity of light(B} \rightarrow \text{D2)\}} \\ \text{3rd term: \{intensity of light(A} \rightarrow \text{D1)\} \times \{intensity of light(B} \rightarrow \text{D2)\}} \\ \text{4th term: \{intensity of light(A} \rightarrow \text{D2)\} \times \{intensity of light(B} \rightarrow \text{D1)\}} \\ \text{5th term: intensity interference} \\ \text{6th term and followings: A and B are independent (i.e., incoherent), thus zero on average.} \end{array} \right.$$

$$\langle E_a E_b^* \rangle = 0$$

(In case of $T = R = 0.5$)

$$= \frac{1}{2}(|E_a|^4 + |E_b|^4) \quad \text{constant}$$

(quantum)

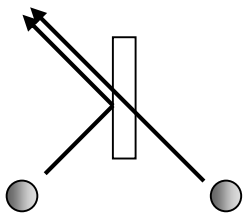
$$\begin{aligned}
 |1\rangle_a \otimes |1\rangle_b &\rightarrow (ir|1\rangle_{d1} + t|1\rangle_{d2}) \otimes (t|1\rangle_{d1} + ir|1\rangle_{d2}) \\
 &= itr|1\rangle_{d1}|1\rangle_{d1} + itr|1\rangle_{d2}|1\rangle_{d2} - r^2|1\rangle_{d1}|1\rangle_{d2} + t^2|1\rangle_{d2}|1\rangle_{d1}
 \end{aligned}$$

The state giving coincident count is $(T - R)|1\rangle_{d1}|1\rangle_{d2}$

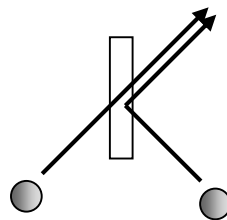
The probability amplitude of this state is zero when $T = R = 0.5$.



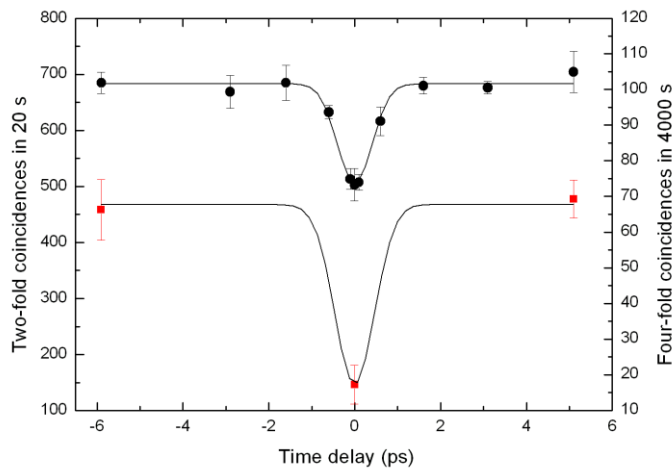
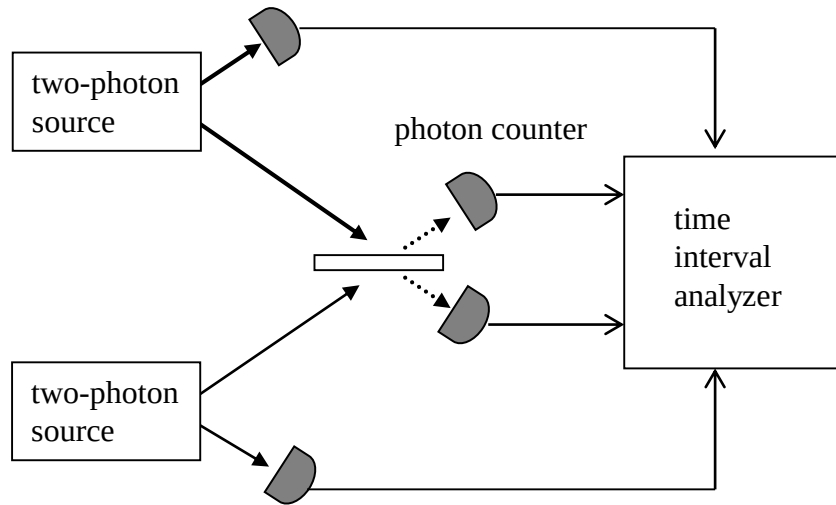
Photons always go to either one of the detectors, not both.



or

*photon bunching*

This phenomenon was confirmed in experimental results.



The difference in classical and quantum worlds originates from the fact that a photon is not divided into two.

If the 1st and 2nd terms in the classical expression do not exist,

$$R^2|E_a|^2|E_b|^2 + T^2|E_a|^2|E_b|^2 - 2TR|E_a|^2|E_b|^2 = (T - R)^2|E_a|^2|E_b|^2 \xrightarrow{(T=R)} 0$$

A two-photon interference occurs, strongly depending on “*indistinguishability of photons*”. When a photon from source A and that from source B have different properties (in terms of, for example, polarization, wavelength, spectral shape, and temporal shape),

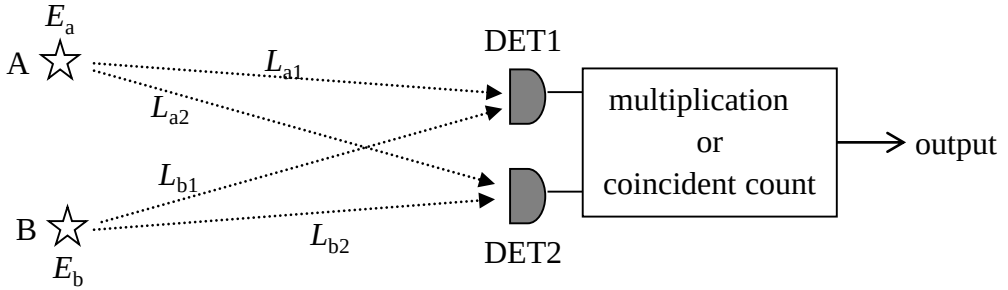
$$\begin{aligned}
 |1\rangle_a \otimes |1'\rangle_b &\rightarrow (ir |1\rangle_{d1} + t |1\rangle_{d2}) \otimes (t |1'\rangle_{d1} + ir |1'\rangle_{d2}) \\
 &= itr |1\rangle_{d1} |1'\rangle_{d1} + itr |1\rangle_{d2} |1'\rangle_{d2} - r^2 |1\rangle_{d1} |1'\rangle_{d2} + t^2 |1\rangle_{d2} |1'\rangle_{d1}
 \end{aligned}$$

Incomplete interference = partially distinguishable

$\neq 0$ (even for $T = R$)

Hanbury-Brown Twiss interference

We detect two lights from different point sources by two detectors, and multiply the detected signals.



(Classical)

(light intensity@D1) $\times$ (light intensity@D2)

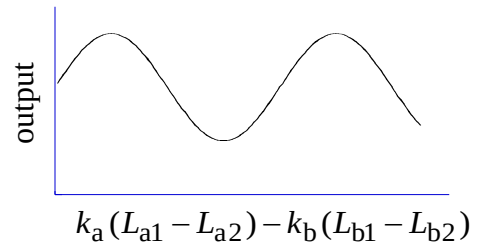
$$\begin{aligned}
 &= \left| E_a e^{ik_a L_{a1}} + E_b e^{ik_b L_{b1}} \right|^2 \times \left| E_a e^{ik_a L_{a2}} + E_b e^{ik_b L_{b2}} \right|^2 \\
 &= \{|E_a|^2 + |E_b|^2 + E_a E_b^* \exp[i(k_a L_{a1} - k_b L_{b1})] + c.c.\} \\
 &\quad \times \{|E_a|^2 + |E_b|^2 + E_a E_b^* \exp[i(k_a L_{a2} - k_b L_{b2})] + c.c.\} \\
 &= |E_a|^4 + |E_b|^4 + |E_a|^2 |E_b|^2 + |E_b|^2 |E_a|^2 + |E_b|^2 |E_a|^2 \exp[i\{k_a (L_{a1} - L_{a2}) - k_b (L_{b1} - L_{b2})\}] + c.c. \\
 &\quad + |E_a|^2 E_a E_b^* \exp[i(k_a L_{a2} - k_b L_{b2})] + \dots \\
 &= |E_a|^4 + |E_b|^4 + |E_a|^2 |E_b|^2 + |E_b|^2 |E_a|^2 + 2|E_a|^2 |E_b|^2 \cos[k_a (L_{a1} - L_{a2}) - k_b (L_{b1} - L_{b2})] + \dots
 \end{aligned}$$

$$\left\{ \begin{array}{l}
 \text{1st term: \{intensity of light(A} \rightarrow \text{D1)\} } \times \text{\{intensity of light(A} \rightarrow \text{D2)\}} \\
 \text{2nd term: \{intensity of light(B} \rightarrow \text{D1)\} } \times \text{\{intensity of light(B} \rightarrow \text{D2)\}} \\
 \text{3rd term: \{intensity of light(A} \rightarrow \text{D1)\} } \times \text{\{intensity of light(B} \rightarrow \text{D2)\}} \\
 \text{4th term: \{intensity of light(A} \rightarrow \text{D2)\} } \times \text{\{intensity of light(B} \rightarrow \text{D1)\}} \\
 \text{5th term: intensity interference} \\
 \text{6th term and followings: A and B are independent (i.e., incoherent), thus zero on average.}
 \end{array} \right.$$

$\langle E_a E_b^* \rangle = 0$

In case of $|E_a| = |E_b|$

$$= 2 |E|^4 \{2 + \cos[k(L_{a1} - L_{a2} - L_{b1} + L_{b2})]\}$$



(Quantum)

One photon is emitted from A and B, respectively.

These two photons are regarded as one quantum state as a whole (i.e., product state), whose initial state is expressed as $|1\rangle_a \otimes |1\rangle_b$ ($|1\rangle_{a(b)}$: one photon at A(B))

Each photon reaches one of the two detectors. The whole state at the detectors is expressed as

$$\begin{aligned} |1\rangle_a \otimes |1\rangle_b &\rightarrow (a_1 e^{ik_a L_{a1}} |1\rangle_{d1} + a_2 e^{ik_a L_{a2}} |1\rangle_{d2}) \otimes (b_1 e^{ik_b L_{b1}} |1\rangle_{d1} + b_2 e^{ik_b L_{b2}} |1\rangle_{d2}) \\ &= a_1 b_1 e^{i(k_a L_{a1} + k_b L_{b1})} |1\rangle_{d1} |1\rangle_{d1} + a_1 b_2 e^{i(k_a L_{a1} + k_b L_{b2})} |1\rangle_{d1} |1\rangle_{d2} \\ &\quad + a_2 b_1 e^{i(k_a L_{a2} + k_b L_{b1})} |1\rangle_{d2} |1\rangle_{d1} + a_2 b_2 e^{i(k_a L_{a2} + k_b L_{b2})} |1\rangle_{d2} |1\rangle_{d2} \\ &= \{a_1 b_2 e^{i(k_a L_{a1} + k_b L_{b2})} + a_2 b_1 e^{i(k_a L_{a2} + k_b L_{b1})}\} |1\rangle_{d1} |1\rangle_{d2} \\ &\quad + a_1 b_1 e^{i(k_a L_{a1} + k_b L_{b1})} |1\rangle_{d1} |1\rangle_{d1} + a_2 b_2 e^{i(k_a L_{a2} + k_b L_{b2})} |1\rangle_{d2} |1\rangle_{d2} \end{aligned}$$

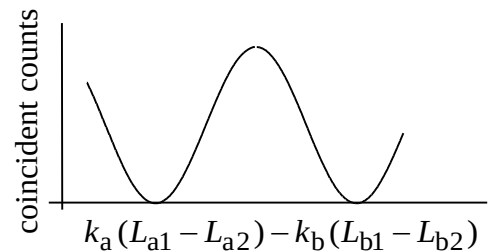
- 1st term: one photon at D1 & one photon at D2
- 2nd term: two photons at D1
- 3rd term: two photons at D2

The probability that each detector counts a photon is given by the 1st term as

$$\begin{aligned} &\left| a_1 b_2 e^{i(k_a L_{a1} + k_b L_{b2})} + a_2 b_1 e^{i(k_a L_{a2} + k_b L_{b1})} \right|^2 \\ &= (a_1 b_2)^2 + (a_2 b_1)^2 + 2a_1 a_2 b_1 b_2 \cos[k_a(L_{a1} - L_{a2}) - k_b(L_{b1} - L_{b2})] \end{aligned}$$

(In case of $a_1 = a_2 = b_1 = b_2$)

$$= 2 |a|^4 \{1 + \cos[k(L_{a1} - L_{a2} - L_{b1} + L_{b2})]\}$$



The extinction ratio is different in classical and quantum interferences.

The difference originates from the fact that a photon is the minimum and thus cannot be divided into two.